Astro 201; Project Set #4

due 3/23/2012

Problems focusing on free-free, bound free, and bound-bound cooling emission; the ionization and excitation state of matter under LTE (Saha/Boltzmann) and non-LTE (collisional ionization equilibrium) conditions.

The Health Benefits of Negative Hydrogen

Until around the 1940's, the primary continuum opacity in the solar atmosphere was unknown. The solution came from an unexpected source. In 1929 Hans Bethe showed that there was a bound state for the negative hydrogen ion, H^-, consisting of one proton and *two* electrons. In 1939, Rupert Wildt made the first suggestion that bound-free opacity from H minus may be important in stellar photospheres, and Chandrasekhar devoted considerable effort to working out the quantum mechanics of the ion[1]. The binding energy of H minus is a mere 0.75 eV; it is a minor constituent of the Sun, but an important one.[2]

Here we'll try to make some very rough estimates of the contributions of various radiative processes to the opacity of the sun. We'll approximate the solar atmosphere as a homogenous medium of pure hydrogen at temperature $T = 5500$ K, number density $n = 10^{17}$ cm^{-3} and scale height $H = 100$ km. We can then make rough estimates of the optical depth of the atmosphere as $\tau \sim n\sigma(\lambda_0)H$, where $\sigma(\lambda_0)$ is the cross-section of some radiative process at a chosen reference wavelength in the optical, $\lambda_0 = 5000$ Å.

[1] e.g., Chandrasekhar 1944

[2] A google search on "negative hydrogen ion" turns up many sites pitching the antioxidant power of the substance. I'm not sure if eating these pills will help you live longer; but at least we demonstrate here that negative hydrogen does have an intimate connection to life – most of the light we receive on earth has passed through the H minus ion. (If the sellers knew this fact, I bet they would include it in their pitch).

a): Assuming LTE, estimate the ionization fraction of the solar atmosphere: $x_{\rm HII} = n_e/n$, where n_e is the electron density. Assume that all of the free electrons come from ionized hydrogen (i.e., ignore H minus and metals for now). What is an estimate for the optical depth to electron scattering?

b): Estimate the optical depth at the reference wavelength to free-free absorption from ionized hydrogen.

c): Estimate the optical depth at the reference wavelength to bound-free absorption from the $n = 1$, $n = 2$, and $n = 3$ states of hydrogen.

You will see that none of the above sources of continuum opacity are sufficient to understand the solar photosphere. The dominant opacity in the optical comes from the negative hydrogen ion.

d) What is the fractional abundance of H^- ions in the solar atmosphere, assuming LTE? Estimate the optical depth (at the reference wavelength) to bound free absorption from the ground state of H^- using a hydrogenic approximation for the cross-section.

Comment: The actual H-minus bound-free cross-section is more complex, due to the more involved quantum mechanics. As seen in Figure 1, the cross-section initially *rises* from threshold, reaching a peak at a frequency a factor of 2 higher (a wavelength $\sim$ 8000 Å), then declines in a typical way[3]. This wavelength dependence of the opacity was known – based on observations of solar limb darkening at various wavelengths – even before Wildt made his suggestion. Much of Chandrasekhar's subsequent work on H minus was thus in an effort to explain the opacity peak at around 8000 Å.

[3] A detailed review of the H minus physics is given in Rau (1996)

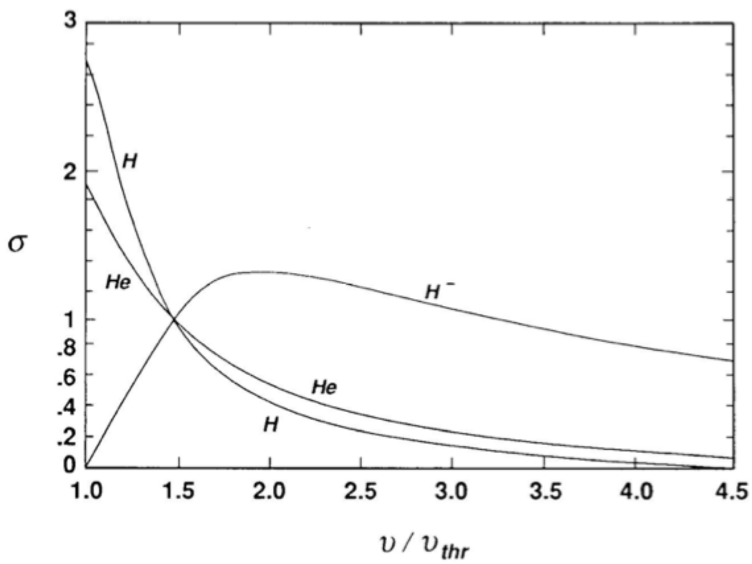

Figure 1: Contrast between the cross-section for photoionization of neutral atoms and photodetachment of a negative ion. Horizontal axis is the incident photon frequency in units of the threshold frequency (from Bethe & Salpeter 1997, reproduced in Rau 1996).

Cooling in the Halo

A VERY SIMPLE PICTURE OF GALAXY FORMATION considers the accumulation of baryonic gas in a dark matter halo. As gas falls into a halo, it is may be shocked to temperatures near the virial temperature, T_v, set by the gravitational potential

$$kT_v \sim GMm_p/R \qquad (1)$$

Where M is the mass of the system and R the radius. If this shock heated gas can cool on a dynamical timescale, it may lose pressure support and condense into a galaxy[4]. The timescale for gravitational collapse is

$$t_{\text{dyn}} \sim \left[\frac{GM}{R^3}\right]^{-1/2} \tag{2}$$

[4] Presumably the contraction will eventually be stopped by the forming of a rotationally-supported disk or by fragmentation into stars.

In the largest halos, the gas will have virial temperatures $T_v > 10^7$ K and hydrogen will be completely ionized. In this case, free-free emission dominates the radiative cooling and we can make some very simple estimates of the size of the largest physical structures that may form galaxies.

a) Assume that gas in a massive halo is of constant density, spherically symmetric, and has a total mass of order the dark matter mass. If the cooling is due entirely from free-free emission from ionized hydrogen, show that the condition that the radiative cooling time is shorter than the dynamical timescale sets an upper limit on the radii of the largest structures that can collapse: $R_g \sim 80$ kpc. This simplistic estimate is actually a pretty reasonable upper limit on the size of massive galaxies in the Universe.

The real dynamics of galaxy formation are of course very complicated, and sophisticated 3-dimensional simulations are needed. Recent studies suggest that not all gas is shocked to the virial temperatures, rather much of the infall comes from narrow streams of cool ($T \sim 10^4$ K) dense gas, which are strung along filaments of the cosmic web of dark matter (see Figure 2). In addition to gravitational infall, one should also consider feedback from a variety of sources (e.g., energy injection from stars, supernovae, and AGN) which may significantly affect the dynamics. But whatever the complexities, a realistic description of radiative cooling is an essential component of any galaxy formation simulation.

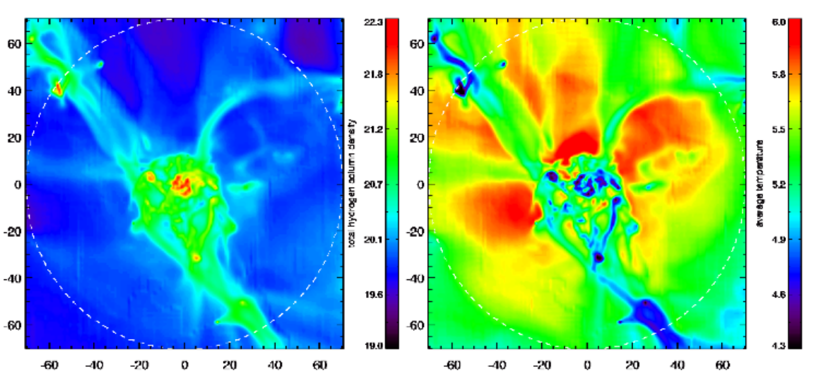

Figure 2: Simulation of a galaxy forming in a dark matter halo at redshift $z = 2.3$. The left plot shows the column density of gas, and the right plot the (density weighted) gas temperature. The dashed circle marks the virial radius of 72 kpc. One notices narrow streams of cool gas feeding the galactic disk (viewed here nearly face on). The streams are imbedded in a lower density medium of hot gas that has been shocked to the virial temperature.

Several papers have calculated and tabulated the cooling of astrophysical plasmas due to the variety of radiative processes (free-free, bound-free, bound-bound emission). A common assumption is that the gas is in collisional ionization equilibrium (CIE) and that it is optically thin, so that any radiation produced escapes without being reabsorbed. The collisional processes that lead to cooling scale as $n_e \times n_H$, where n_e is the electron number density and n_H is the hydrogen gas density. Thus the power emitted per unit volume can be written

$$\epsilon(T) = n_e n_H \Lambda(T) \tag{3}$$

where $\Lambda(T)$ (units ergs s^{-1} cm^3) is a volumetric cooling function (or cooling coefficient). The figure in the margin shows results from one frequently referenced study. Because such cooling functions are quite important and invoked in a variety of astrophysical simulations, let's try to understand and reproduce the general features of the curve.

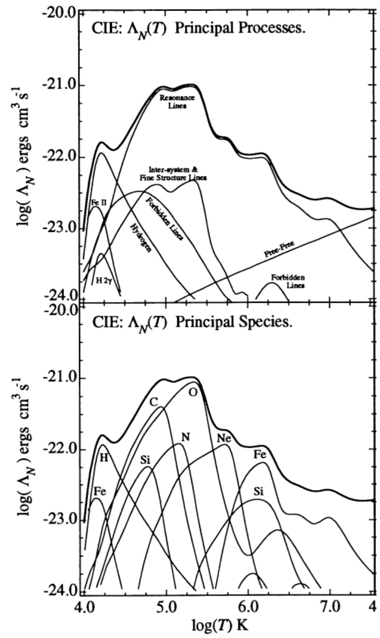

Figure 3: Cooling function for low density astrophysical plasma from Sutherland and Dopita (1993) . The top panel shows the contributions from the different radiative processes, while the bottom panel shows the contributions from different elements.

b) First, consider gas composed of pure hydrogen. Under the assumption of CIE, calculate the contributions to $\Lambda(T)$ from (1) free-free emission (2) bound-free emission (3) collisionally excited Lyman alpha line emission. Look at the temperature range, $T = 10^4 - 10^8$ K and plot (on a log-log scale like Figure 3) the cooling functions for each of the three contributions. You can use the (very) approximate collisional ionization and excitation and radiative recombination rates provided on the website .

c) Our optically thin assumption is reasonable for free-free and (some) bound-free emission, but can easily fail for Lyman alpha line photons[5]. However, the probability that a photon is actually reabsorbed in the L_α line is very small. Write down (in terms of the Einstein coefficients C_{21} and A_{21}) the probability that a L_α photon is absorbed into the thermal pool during a single line interaction. Show that for $n \sim 1$ cm^{-3}, $T \sim 10^4$ K, the probability of such absorption is small. The assumption of optically thin line cooling is therefore usually OK – the emitted L_α photons may scatter in the line many times, but will eventually escape the system without ever really reheating the gas.

[5] You should be able to easily show that the optical depth at line center for Lyman alpha can be as large as $\tau \sim 10^{10}$ for a gas cloud of radius 100 kpc, density 1 cm^{-3}, and assuming that some significant fraction of the gas remains cool $T \sim 10^4$ K and neutral.

Your pure hydrogen calculation gives decent results for the lowest and highest temperatures considered, but it is clear from the Sutherland and Dopita figure that emission from metals dominates in the intermediate range $T \sim 10^5 - 10^6$ K. In particular, collisionally excited emission from resonance lines (i.e., lines where the lower level is the ground state) of metals is very important. This suggests that the dynamics of cooling systems may be sensitive to metallicity.

Rather than try to reproduce the cooling function for all metals, let's just consider one of the most important species, oxygen, emitting from the ionization state OVI. We'll continue to assume that collisional ionization and radiative recombination dominate the bound-free transition rates.

d) To determine the fractional abundance of OVI in the plasma, consider the bound-free transitions between the three ionization states, OV, OVI, and OVII (assume for now that the abundances of all other ionization stages of oxygen are zero). The relevant rates are thus the collisional ones (C_{56}, C_{67}) and the radiative ones (R_{65}, R_{76}) where, e.g., C_{56} is the collisional ionization rate from OV to OVI. Write down an analytic expression for the fraction of OVI in terms of these four rates.

e) Go to the NIST website and have it make a Grotrian diagram for OVI. Identify the wavelength of the one resonance line that you think will be most important for cooling. Take a look at the Grotrian diagrams for other ionization stages of oxygen and argue that resonant line cooling from OVII and OVIII will likely be less important, but that line cooling from lower ionization states like OV will likely make a significant contribution.

f) Calculate the cooling function from collisional excited emission of the OVI resonance line found in **e)** and add it to your plot. In doing this calculation you can assume that hydrogen is fully ionized so that $n_e = n_H$. Take the metallicity to be solar.

Comment Your cooling function should now be one step closer to the one of Sutherland and Dopita, and you can imagine adding in the contributions of other metals and ionization states to fill in the curve. Naturally, the published curves use more accurate expressions for the transition rates than we have, and include a more complete list of lines. The Sutherland and Dopita paper and other papers also consider deviations from CIE due to photoionization, which is often an important factor in galaxy formation.

Comment You can see from your cooling curve that our assumption in part **a)** that free-free cooling dominates is OK for $T > 10^7$ K, which is appropriate for the shocked gas in the most massive systems. But if we have cooler, unshocked gas in the halo, or are considering less massive galaxies with $T_v < 10^7$ K, it will be important to include other radiative processes. The narrow streams of cool dense gas seen in Figure 2, for example, have $T \sim 10^4$ K and so should be effective Lyman alpha line emitters. In fact, looking for line emission from the circumgalactic medium is one good way to test the predictions of these simulations. Observers have found many examples of extended ($\sim$ 100 kpc) L_α emission nebula at redshifts of $z \sim 2$ and beyond. Some have suggested that these so-called "Lyman alpha blobs" represent the cool, infalling gas seen in the simulations, although this identification is still debated.

Comment Somewhat similar arguments about gravitational infall and cooling appear in modeling the formation of molecular clouds and stars. Our cooling curve so far is only reasonable for temperatures around 10^4 K or above. At lower temperatures, cooling from dust and molecular line emission may start to play a significant role. Note also that if the system becomes dense enough, the assumption of optically thin cooling may no longer hold, and one would have to solve the radiative transfer equation to determine the rate at which radiation actually escapes the system. In 3-D simulations, this is often done using the flux-limited diffusion approximation.